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$$H = \begin{bmatrix} I & I & I & \dots & I \\ I & \sigma^1 & \sigma^2 & \dots & \sigma^{p-1} \\ I & \sigma^2 & \sigma^4 & \dots & \sigma^{2(p-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & \sigma^{r-1} & \sigma^{2(r-1)} & \dots & \sigma^{(r-1)(p-1)} \end{bmatrix}$$

FIG.1

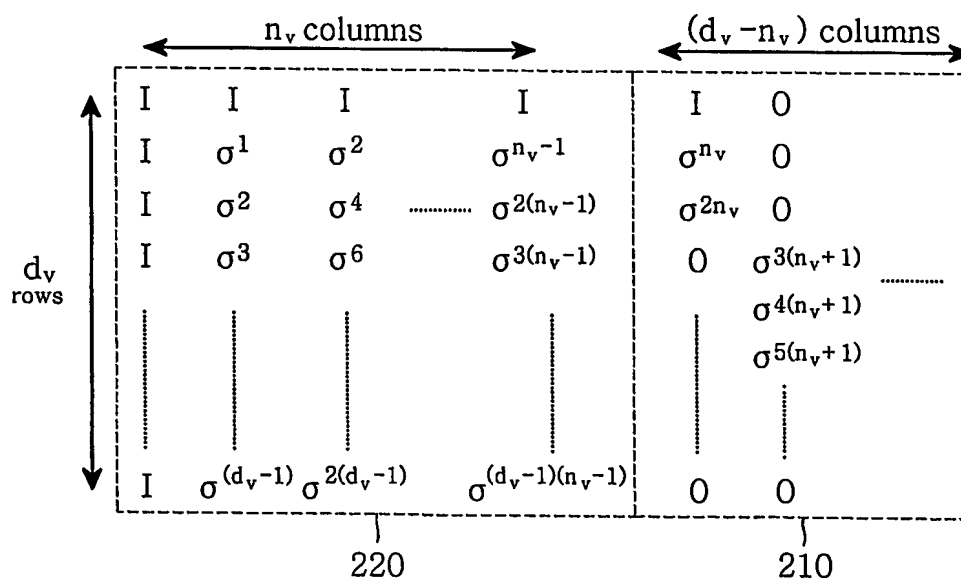


FIG.2

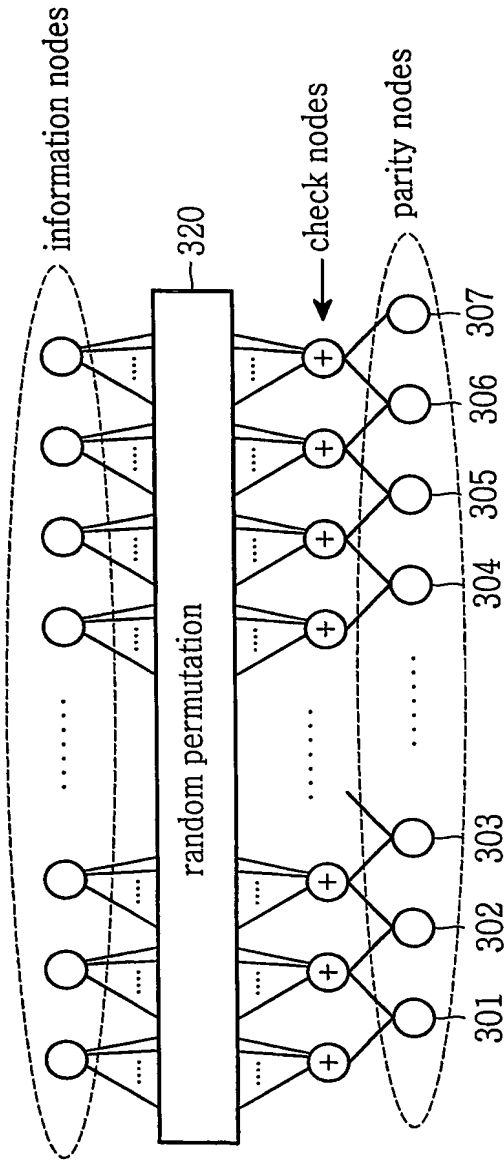


FIG.3

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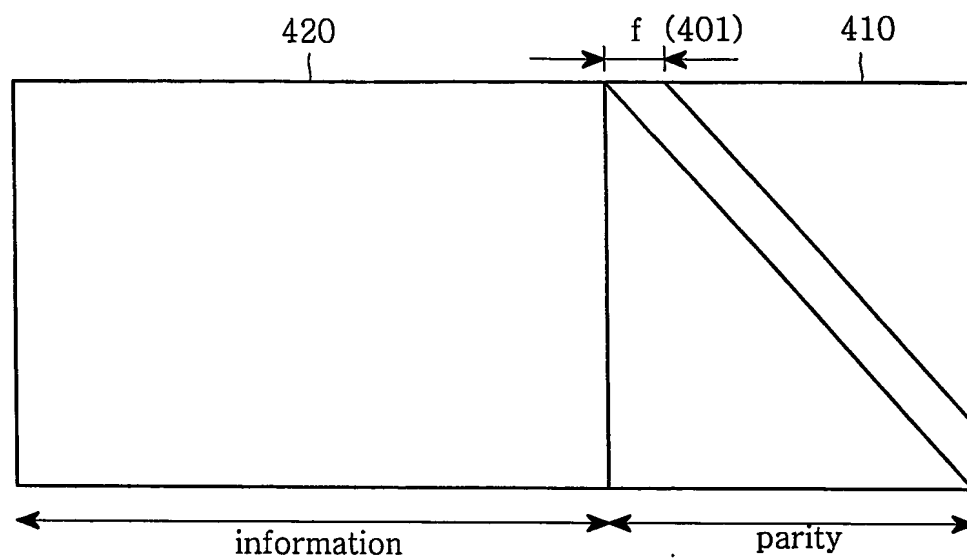


FIG.4

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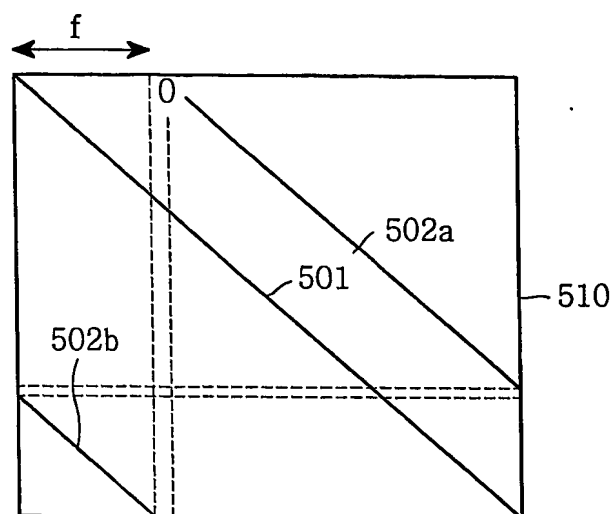


FIG. 5

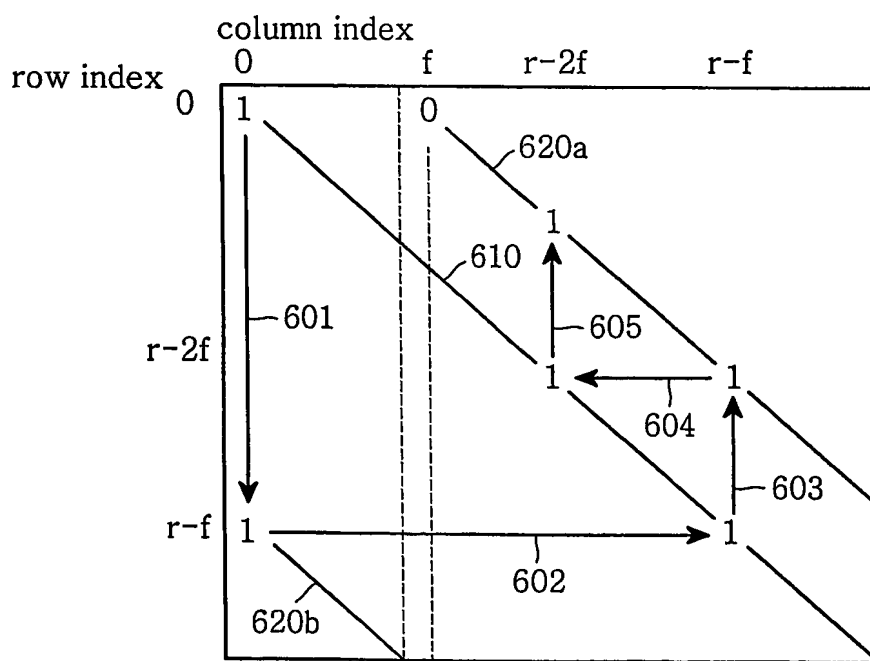


FIG. 6

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0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28			48
0	5	10		20		35			60
0	6	12		24		42			72
0	7	14		28			56		2
0	8	16		32			64		15
0	9	18		36			72		28
0	10	20			50			1	51
0	11	22			55			10	65
0	12	24			60			19	79
0	13	26			65				
0	14	28			70			41	
								51	

Example of H_d with irregular distribution of $d_v = 15$ ($p=89$)

FIG.7

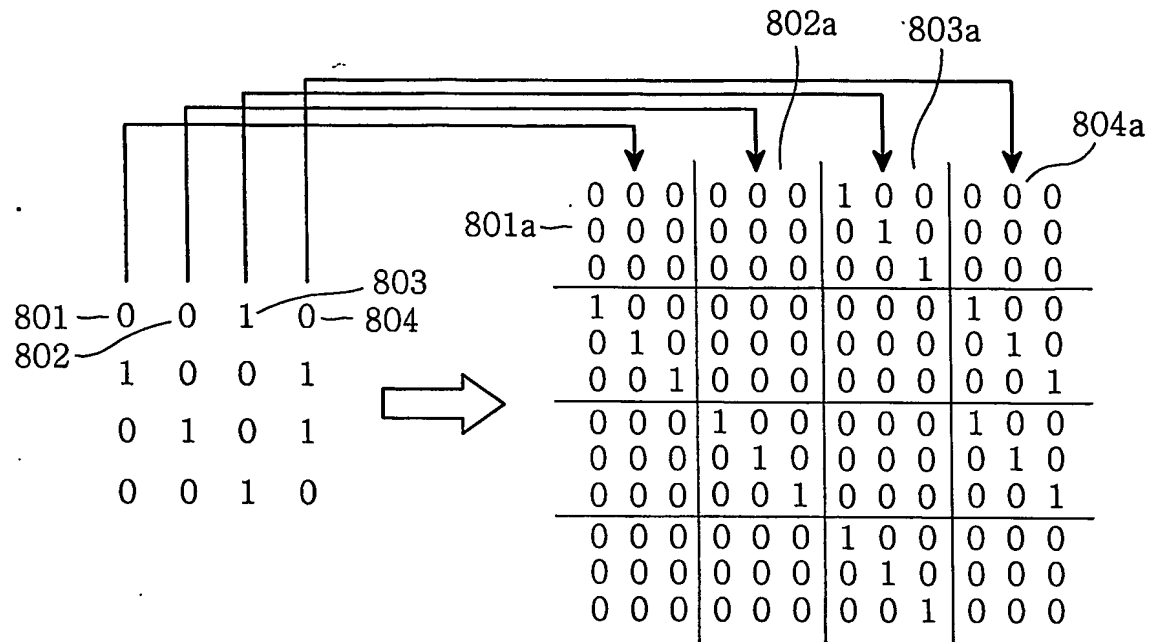


FIG.8

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$$\begin{array}{c}
 \text{fsub-matrix columns(pxf columns)} \\
 \longleftrightarrow \\
 \begin{array}{cc}
 \sigma^{j_0} & \sigma^{j_1} \\
 \sigma^{j_2} & \sigma^{j_3} \\
 & \vdots \\
 & \sigma^{j_{2(r-f-1)}} \quad \sigma^{j_{2(r-f-1)+1}} \\
 \sigma^{j_{2(r-f)+1}} & \sigma^{j_{2(r-f)}} \\
 & \vdots \\
 & \sigma^{j_{2(r-1)+1}} \quad \sigma^{j_{2(r-1)}}
 \end{array}
 \end{array}
 \quad H_P = \left[\begin{array}{c} \text{matrix structure} \end{array} \right]$$

FIG.9

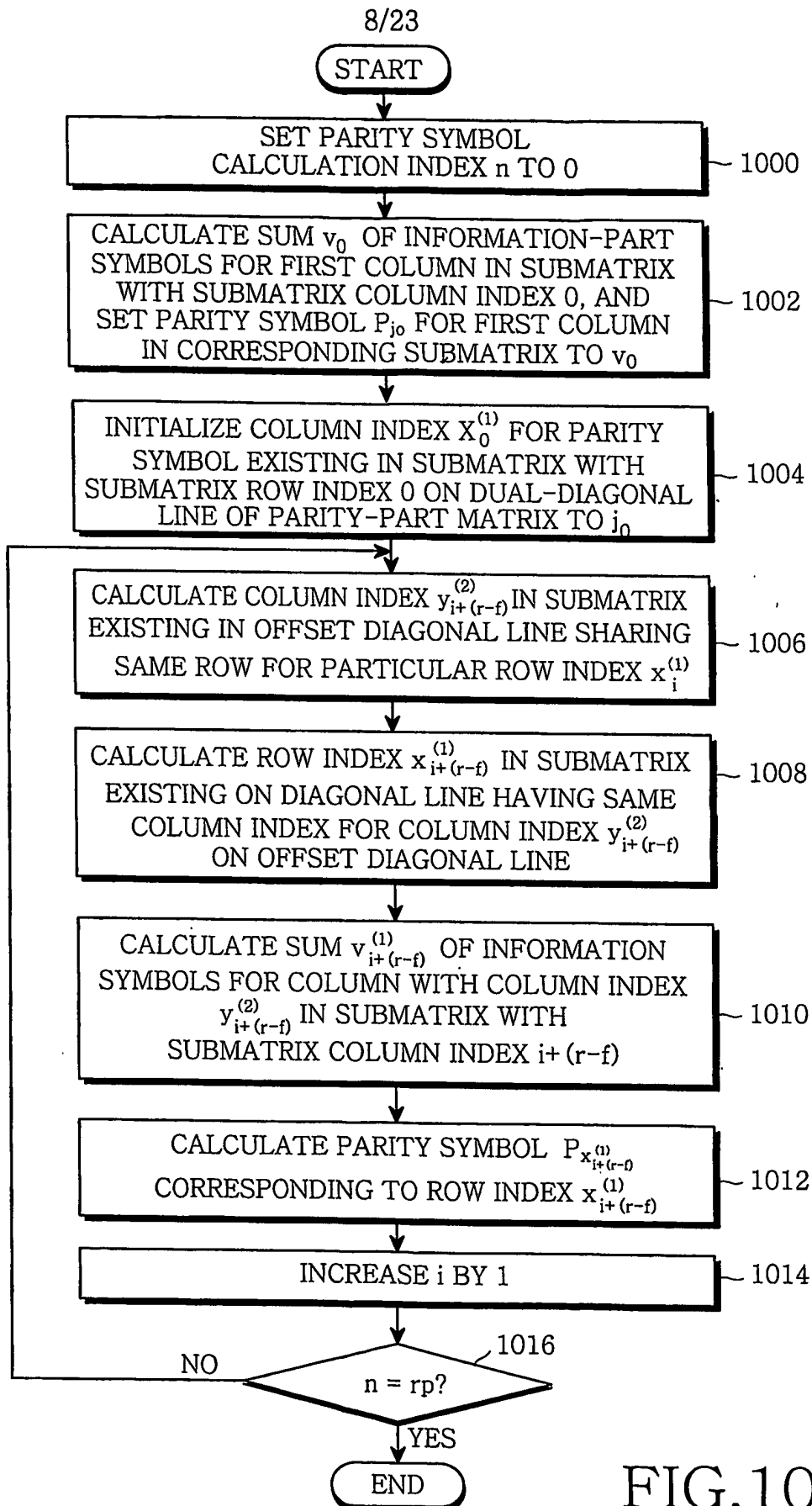
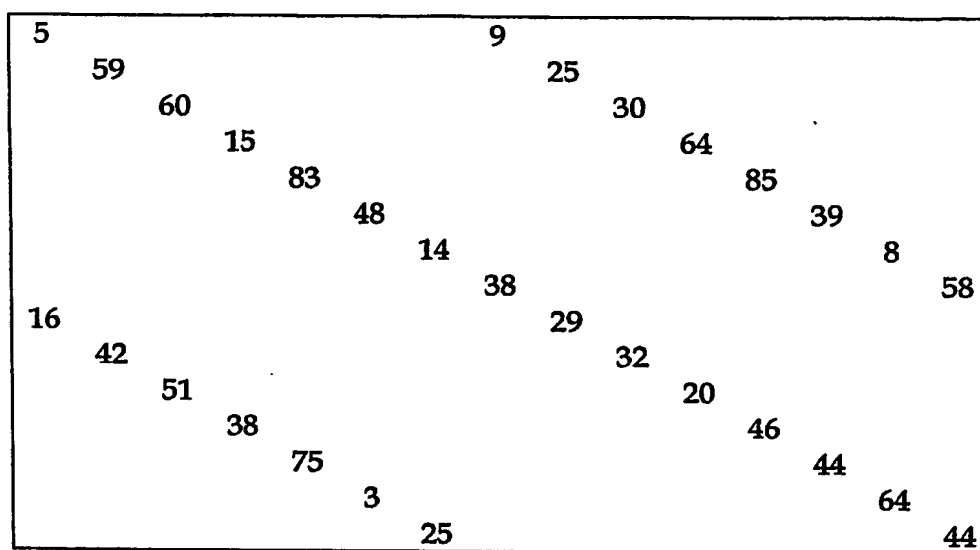


FIG.10



Example of H_p by lifting the generalized dual-diagonal matrix ($r=15, f=7, p=89$)

FIG.11

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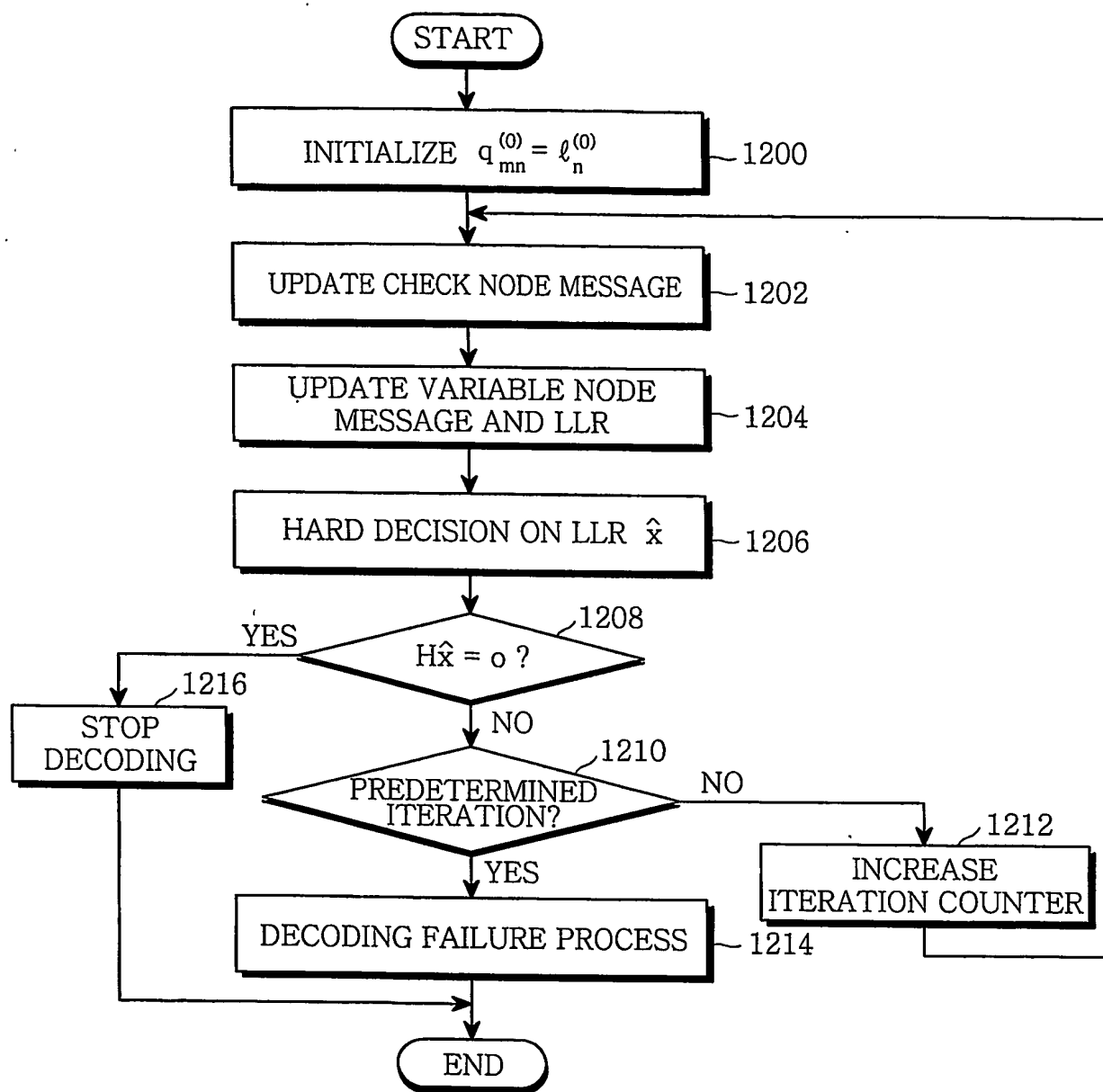


FIG.12

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- H_d

0	0	0	0		0		0	
0	1	2	3		6		11	
0	2	4	6		12		22	
0	3	6	9		18		4	
0	4	8	12		28		19	
0	5	10		20	6		2	
0	6	12		24	13		14	
0	7	14		28		27		4
0	8	16		3		6		17
0	9	18		7		14		1
0	10	20		21		3		24
0	11	22		26		12		9
0	12	24		2		21		23
0	13	26		7		14		
0	14	28		12		24		

FIG.13A

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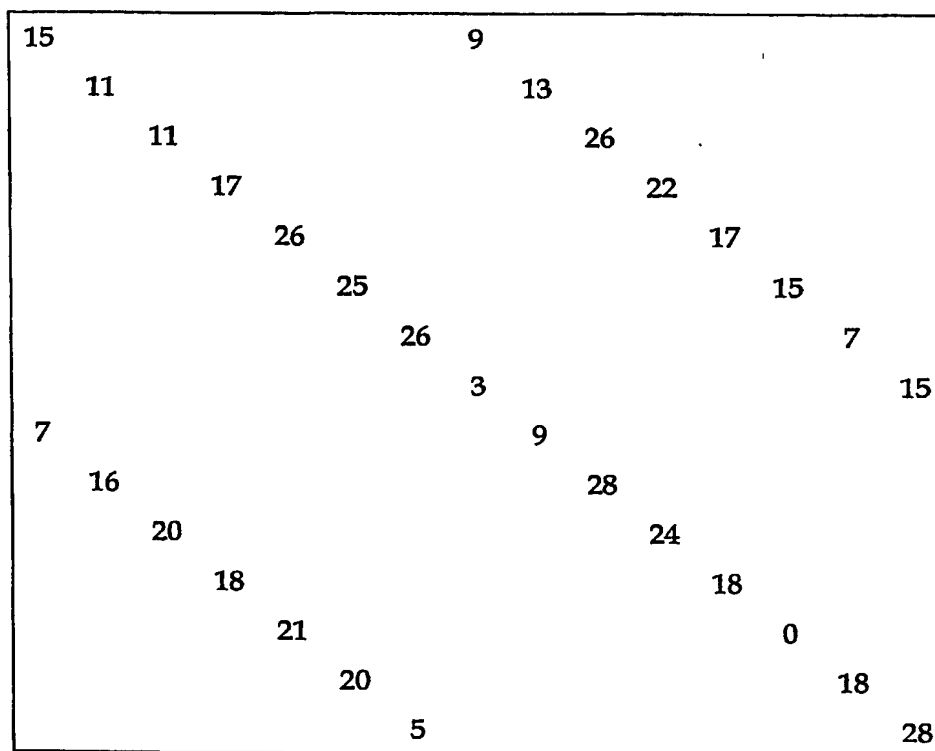
 $-H_p$ 

FIG.13B

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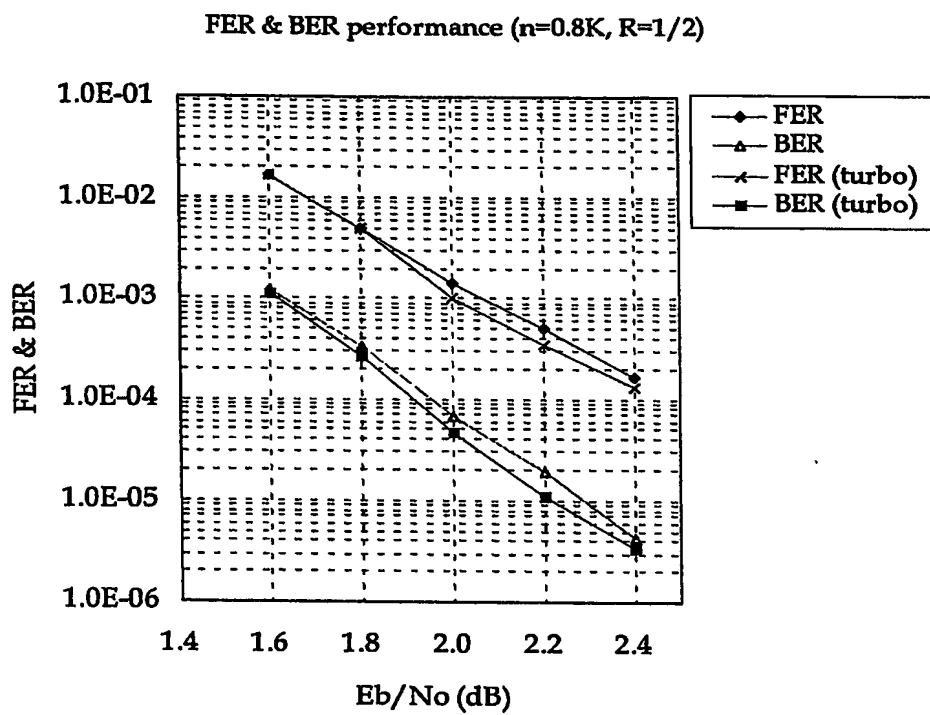


FIG.13C

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- H_d

0	0	0	0		0		0		
0	1	2	3		6		11		
0	2	4	6		12		22		
0	3	6	9		18		33		
0	4	8	12			28		48	
0	5	10		20		35		7	
0	6	12		24		42		19	
0	7	14		28		3			38
0	8	16		32		11			51
0	9	18		36		19			11
0	10	20			50		37		34
0	11	22			2		46		48
0	12	24			7		2		9
0	13	26			12			24	
0	14	28			17			34	

FIG.14A

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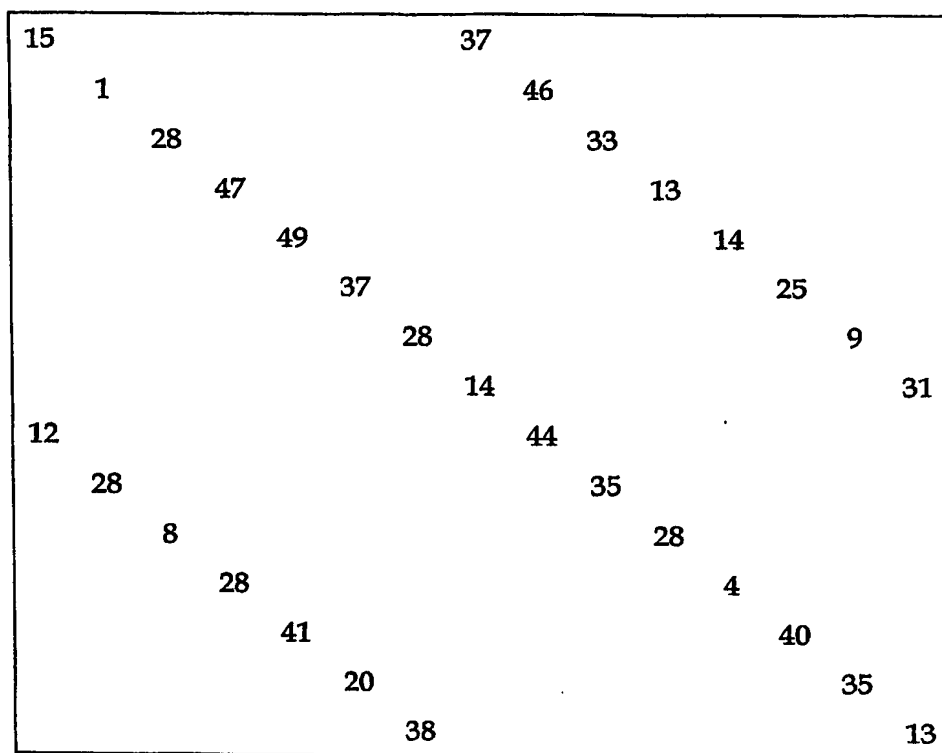
 $-H_p$ 

FIG.14B

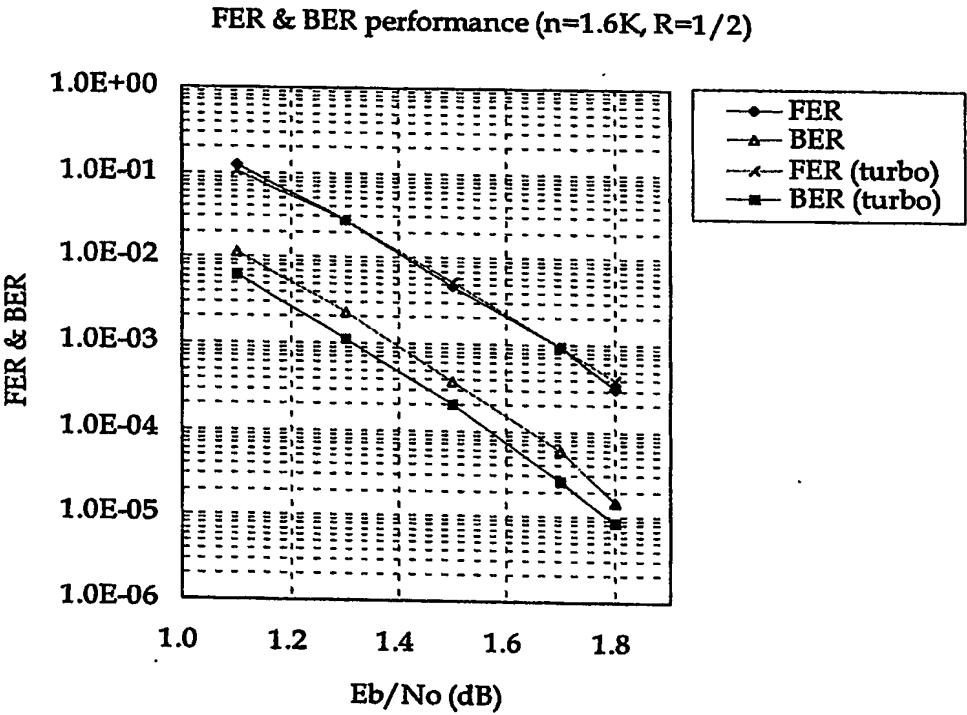


FIG.14C

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- H_d

0	0	0	0		0		0	
0	1	2	3		6		11	
0	2	4	6		12		22	
0	3	6	9		18		33	
0	4	8	12		28		48	
0	5	10		20	35		60	
0	6	12		24	42		72	
0	7	14		28		56		91
0	8	16		32		64		1
0	9	18		36		72		14
0	10	20		50		90		37
0	11	22		55		99		51
0	12	24		60		5		65
0	13	26		65		27		
0	14	28		70		37		

FIG.15A

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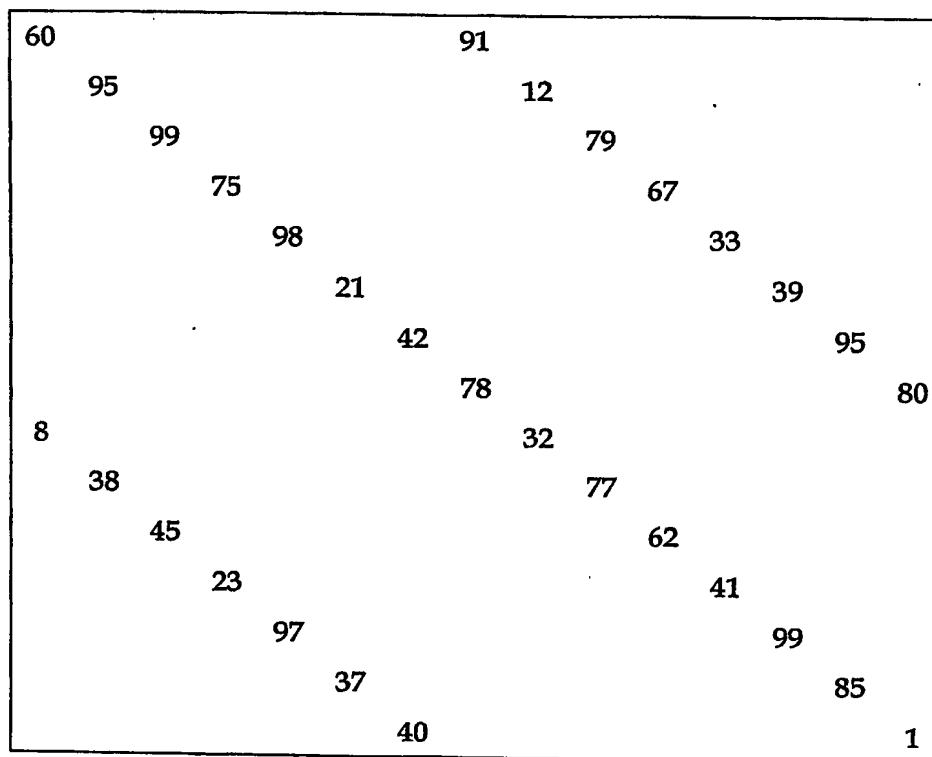
- H_p

FIG.15B

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FER & BER performance (n=3.1K, R=1/2)

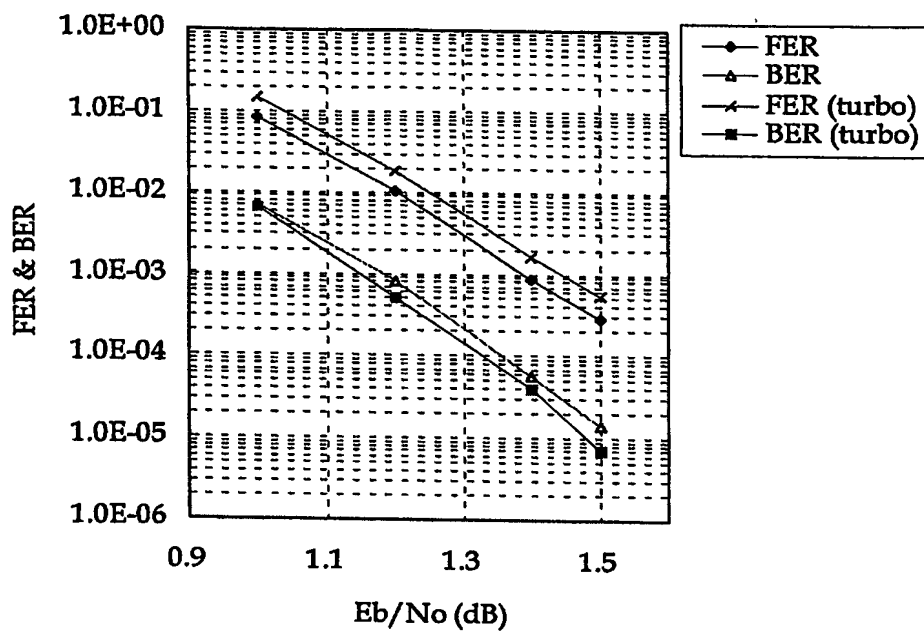


FIG.15C

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- H_d

0	0	0	0		0		0	
0	1	2	3		6		11	
0	2	4	6		12		22	
0	3	6	9		18		33	
0	4	8	12			28		48
0	5	10		20		35		60
0	6	12		24		42		72
0	7	14		28			56	91
0	8	16		32			64	104
0	9	18		36			72	117
0	10	20			50		90	140
0	11	22			55		99	154
0	12	24			60		108	168
0	13	26			65			130
0	14	28			70			140

FIG.16A

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28			48
0	5	10		20		35			60
0	6	12		24		42			72
0	7	14		28			56		
0	8	16		32			64		2
0	9	18		36			72		15
0	10	20			50				28
0	11	22			55		1		51
0	12	24			60		10		65
0	13	26			65		19		79
0	14	28			70			41	
								51	

Example of H_d with irregular distribution of $d_v = 15$ ($p=89$)

FIG.16B

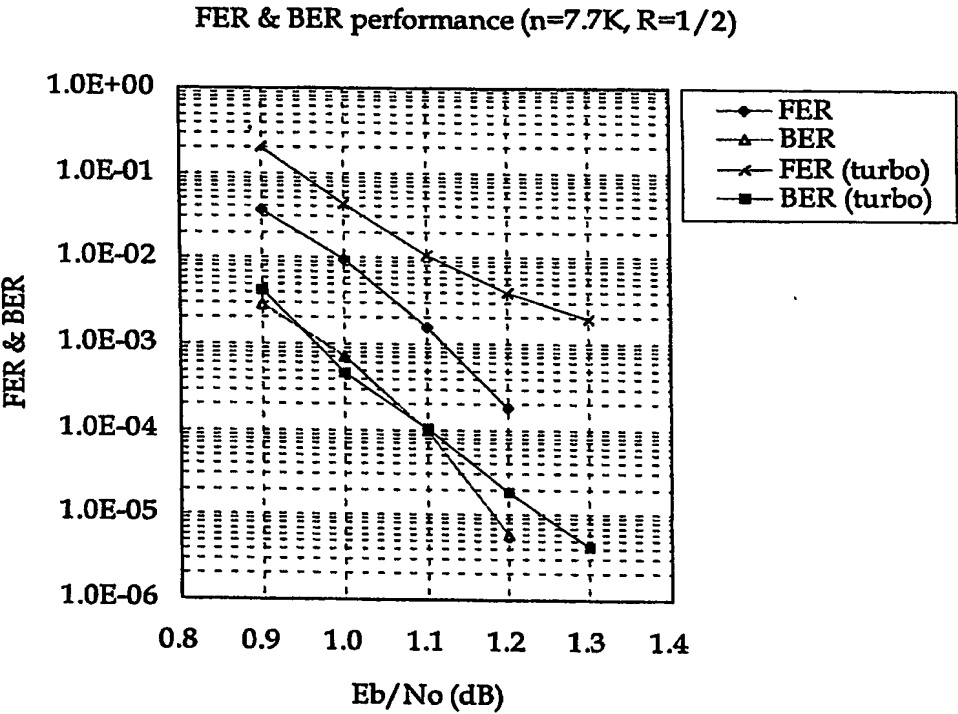


FIG.16C

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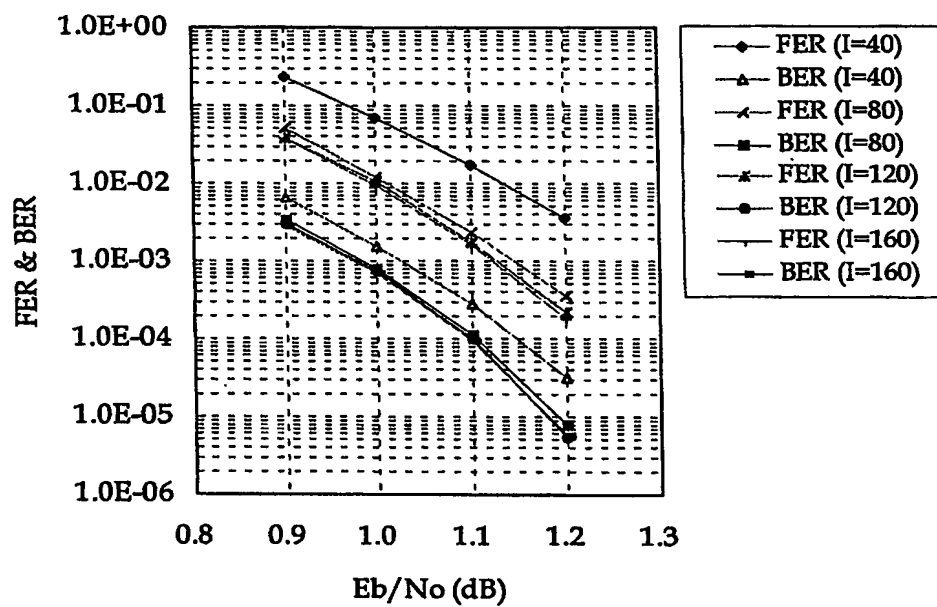
FER & BER performance ($n=7.7K$, $R=1/2$)

FIG.16D